Integration is a mathematical technique which is used to find something whose rate of change is known. In 17th century Newton and Leibnitz discovered the idea of integration. It has a wide range application in engineering, medicine, architecture, economics, etc. The objectives of this chapter are to discuss integration and provide standard integration techniques.

**Learning Outcomes**: By the end of this course, students will be able to........

(a). find displacement from velocity and velocity from acceleration.

(b). calculate areas under curves, volumes of solids, arc lengths.

(c). evaluate center of mass, moment of inertia.

(d). determine work done by a force, electric charge etc.

***Integration***: The process of finding an anti-derivative or integral of a function is called integration. It is the inverse process of differentiation. If  be a function of  related with another function in such a way that



then



which is called an indefinite integral of  with respect to .

where,and  are called integrand, integral and constant of integration respectively.

And



which is called the definite integral of  from  to , and ‘’ is called the lower limit and ‘’ the upper limit of the definite integral.

***Fundamental Properties***:

1. .
2. 

where is a constant.

***Integration Formulas***:

1. . 2. .
2. . 4. .

5. . 6. .

1. . 8. .
2. . 10. .
3. . 12. .
4. . 14. .
5. . 16. .
6. . 18. .
7. . 20. .
8. . 22. .
9. . 24. .
10.  26. 

27.. 28. .

1. .
2. .
3. .
4. .
5. . 34. .

35..

36. .

***Illustrative Examples***:

**Problem-01**: **Exercise-01:** .

 Ans: 







where is an integrating constant.

**Problem-02**: **Exercise-02**: 

 Ans: 





where is an integrating constant.

**Problem-03:** **Exercise-03**: 

 Ans: .







where is an integrating constant.

**Problem-04:** **Exercise-04**: 

 Ans: .















whereis an integrating constant.

**Problem-04**:  **Exercise-04**: .

 Ans:.









whereis an integrating constant.

**Problem-05**:  **Exercise-05**:.

 Ans:.









where is an integrating constant.

**Problem-06**:  **Exercise-06**:.

 Ans:.













whereis an integrating constant.

**Problem-07**: **Exercise-07**: .

Ans:.

















whereis an integrating constant.

**Problem-08**: **Exercise-08**:

Ans: 











whereis an integrating constant.

**Problem-09**: 











whereis an integrating constant.

**Problem-10**: **Exercise-09**: 

Ans: .

















whereis an integrating constant.

**Problem-11**: 













whereis an integrating constant.

**Problem-12**: **Exercise-10**: .

Ans:.













where is an integrating constant.

**Problem-13:****Exercise-11:**

Ans: .

















.

whereis an integrating constant.

**Problem-14:****Exercise-12:**

Ans:















whereis an integrating constant.

**Problem-15:****Exercise-13:**

Ans:











whereis an integrating constant.

**Problem-16:**













whereis an integrating constant.

***Method of substitution***

Sometimes, the integration of given integral  is relatively difficult. In this case, we can replace  by  and  by for integrating easily. This process is known as method of substitution.

***Illustrative Examples:***

**Problem-01**: **Exercise-01**: 

Ans: .

put



Now 







where is an integrating constant.

**Problem-02:****Exercise-02:**

Ans: .

put

Now 













where is an integrating constant.

**Problem-03:****Exercise-03:**

Ans: .

put

Now 





where is an integrating constant.

**Problem-04:****Exercise-04:**

Ans: .

put

Now 







where is an integrating constant.

**Problem-05:****Exercise-05:**

Ans: .



put

Now 





where is an integrating constant.

**Problem-06:**











where is an integrating constant.

**Problem-07:****Exercise-06:**

Ans: .

put

Now 









where is an integrating constant.

**Problem-08:****Exercise-07:**

Ans: .

put

Now 













where is an integrating constant.

**Problem-09:****Exercise-08:**

Ans: .

put

Now 







.

where is an integrating constant.

**Problem-10:**







put

Now 







where is an integrating constant.

**Problem-11:**











where is an integrating constant.

**Problem-12:****Exercise-09:**

Ans: 









where is an integrating constant.

**Problem-13:****Exercise-10:**

Ans: 









where is an integrating constant.

**Problem-14:**







put

Now 





where is an integrating constant.

**Problem-15:**











where is an integrating constant.

**Problem-16:****Exercise-11:**

Ans: .

put

Now 



















where is an integrating constant.

**Problem-17:**



put

Now 

















.

where is an integrating constant.

**Problem-18:**





Put

Now 





where is an integrating constant.

**Problem-19:****Exercise-12:**

Ans: 









Put 

Now 





where is an integrating constant.

**Problem-20:**















put

Now 





.

where is an integrating constant.

**Problem-21:****Exercise-13:**

Ans: 



Now 











where,

Put 

Now 





and

Put 

Now 





From (1) we have,



where is an integrating constant.

**Problem-22:****Exercise-14:**

Ans: 



put

or, 



Now 













where is an integrating constant.

***NOTE:*** Integrals of the type  can be evaluated exactly in the same way.

**Some Important Standard Integrals**

**Problem-01:****Exercise-01:**

Ans: .

**Exercise-02:**

Ans: .

**Exercise-03:**

Ans: .

where is an integrating constant.

**Problem-02:**

















where is an integrating constant.

**Problem-03:****Exercise-04:**

Ans: .

**Exercise-05:**

Ans: .





where is an integrating constant.

**Problem-04:****Exercise-06:**

Ans: .







.

where is an integrating constant.

**Problem-05:****Exercise-07:**





















where is an integrating constant.

**Problem-06:****Exercise-08:**











where is an integrating constant.

***NOTE:*** Integrals of the type can be evaluated exactly in the same way.

**Problem-07:****Exercise-09:**

Ans: .

Put 

Now 







.

where is an integrating constant.

**Problem-08:****Exercise-10:**

Ans: .

Put  ; 



Now 





.

where is an integrating constant.

**Problem-09:****Exercise-11:**

Ans: .

Put  ; 

**Exercise-12:**

Now 

 ; putting









where is an integrating constant.

***NOTE:*** Integrals of the type  can be evaluated exactly in the same way.

**Problem-10:** **Exercise-13:**

Ans: .

Put  or **Exercise-14:** 

Ans: .

Now **Exercise-15:** 

Ans: .







where is an integrating constant.

**Problem-11:** **Exercise-16:** 

Ans: .

Put  or 



Now 









where is an integrating constant.

***NOTE:*** Integrals of the type  can be evaluated exactly in the same way.

**Problem-12:** **Exercise-17:** 

Ans: .

Put 



Now 



Again let 

















where is an integrating constant.

***NOTE:*** Integrals of the type  can be evaluated exactly in the same way.

**Problem-13:** **Exercise-18:**

Ans: .

Put **Exercise-19:**

Now Ans: .

**Exercise-20:**





















.

where is an integrating constant.

***NOTE:*** Integrals of the type  can be evaluated exactly in the same way.

***Integration by Parts***

The formula for the integration of a product of two functions is referred to as integration by parts. *i.e,*

.

While applying the above rule for integration by parts to the product of two functions, care should be taken to choose properly the first function, i.e., the function not to be integrated.

**Illustrative Examples**:

**Problem-01:****Exercise-01:**

Ans: 

**Exercise-02:** 

Ans: 



whereis an integration constant.

**Problem-02:**























whereis an integration constant.

**Problem-03:****Exercise-03:**

**Ans:** 









where is an integration constant

**Problem-04:****Exercise- 04:**

**Ans:**













whereis an integration constant.

**Problem-05:****Exercise- 05:**

**Ans:**















where is an integration constant.

**Problem-06:****Exercise-06:**

Ans: 

Put 

Now 











whereis an integration constant.

**Problem-07:** 



















whereis an integration constant.

**Problem-08:**



Put 

Now 











whereis an integration constant.

**Problem-09:Exercise-07: **

**Ans: .**

****

****

****

****

****

****

****

whereis an integration constant.

**Problem-10: Exercise-08: **

Ans: 





















where c is an integrating constant.

***Integration of Trigonometric Functions***

**Problem-01:  Exercise-01: **

**Ans: **







put****

Now





where c is an integrating constant.

**Problem-02:  Exercise-02: **

**Ans: **







put

Now 







where c is an integrating constant.

**Problem-03: Exercise-03: **

**Ans: **

****

****

****

****

put

Now 











where c is an integrating constant.

**Problem-04: **

****

put

Comparing coefficient of ,  and constant terms, we get



Solving,



Now ****

****

****

****

where c is an integrating constant.

**Exercise-04: **

**Ans: **

**Exercise-05: **

**Ans: **

**Multiple choice questions:**

1. Which one of the following is answer of ****

** **

** **

1. The value of the integral ****is :

** **

** **

1. Calculate ****

** **

** **

1. Evaluate ****

** **

** **

1. Find the indefinite integral ****

** **

** **

1. What is the value of ****

** **

** **

1. Evaluate the indefinite integral ****

** **

** **

1. Calculate ****

** **

** **

1. Determine the value of ****

** **

** **

1. Evaluate the value of ****

** **

** **

|  |
| --- |
|  |

**Example-01:** A car starts from rest at  from the origin and has acceleration at time t given by  **​​.** Find the velocity and displacement of the car at

**Solution:** The velocity is,

when then so

Putting value of c in eq.(1) we get,

At the velocity is,

Again the displacement is,

when then so

Putting value of s in eq.(3) we get,

At the displacement is,

**Example-02:** The electric current (in mA) in a computer circuit as a function of time is . What total charge passes a point in the circuit in ?

**Solution:** The amount of charge is,

when then so

Putting value of c in eq.(1) we get,

At the charge is,